A Note on the Generation of Twisting "D" Metrics from Non-Twisting "D" Metrics

T. C. Basey

Dept. of Applied Mathematics, University College of Wales, Aberystwyth *

(Z. Naturforsch. **30 a**, 1200 [1975]; received April 8, 1975)

Using a method of complexification of coordinates it is shown how certain of the Twisting "D" metrics may be obtained from corresponding Non-Twisting "D" metrics.

Newman and Janis [1] were able to show how one could obtain the Kerr [2] metric from the Schwarzschild metric using a certain complex coordinate transformation of the metric variables. Kerr, in a later paper, with Demianski [3] was able to show that a combined Kerr-NUT solution could also be produced using the same method. The same procedure will be used here to generate the Twisting "D" metrics of Kinnersley [4] from Non-Twisting "D" metrics. The following type "D" metrics are complexified:

$$\mathrm{d}s^2 = -(1+(2\,m/r))\cdot \mathrm{d}u^2 + 2\,\mathrm{d}u\,\mathrm{d}r - r^2\,(\mathrm{d}x^2 + \cosh^2 x\,\mathrm{d}y^2) \quad \ldots (1)$$

$$\mathrm{d}s^2 = -(1+(2\,m/r))\cdot \mathrm{d}u^2 + 2\,\mathrm{d}u\,\mathrm{d}r - r^2\,(\mathrm{d}x^2 + \sinh^2 \mathrm{x}\,\mathrm{d}y^2) \quad \ldots (2)$$

$$\mathrm{d}s^2 = -\left(1+\frac{2\,m}{r}\right)\mathrm{d}u^2 + 2\,\mathrm{d}u\,\mathrm{d}r - r^2\,(\mathrm{d}x^2 + e^{2x}\,\mathrm{d}y^2) \quad \ldots (3)$$

$$\mathrm{d}s^2 = -\frac{(x^2+2\,m\,r)}{r^2+\frac14 x^4}\,\mathrm{d}u^2 - (r^2+\tfrac14 x^4)\,\mathrm{d}x^2 - \left\{\frac{m\,r\,x^8}{8(r^2+\frac14 x^4)} + \frac{x^2\,r^4}{(r^2+\frac14 x^4)}\,\mathrm{d}y^2\right\} + 2\,\mathrm{d}u\,\mathrm{d}r + \frac{2\,x^2}{r^2+\frac14 x^4}\,(x^2-\tfrac12\,m\,r\,x^2)\cdot \mathrm{d}u\,\mathrm{d}y + \tfrac12 x^4\,\mathrm{d}r\,\mathrm{d}y\,. \quad \ldots (4)$$

$$\mathrm{d}s^2 = -2\,c\,r^2 x^{-2}\,\mathrm{d}u^2 + 2\,\mathrm{d}u\,\mathrm{d}r - 4\,r\,x^{-1}\,\mathrm{d}u\,\mathrm{d}x - \frac{\mathrm{d}x^2}{2(c+m/x)} - \frac{2\,\mathrm{d}y^2}{(c+m/x)}\,. \quad \ldots (5)$$

The metrics are written in complex null-tetrad form allowing the radial coordinate, r, and the constant, m, to take complex values. In complex null tetrad form metri c (1) becomes:

$$m_\mu = \delta_\mu{}^2\,\frac{1}{\sqrt2}\,(-r) - \frac{i}{\sqrt2}\cosh x\,\delta_\mu{}^3\,r\,,$$

$$\bar m_\mu = \delta_\mu{}^2\,\frac{1}{\sqrt2}\,(-\bar r) + \frac{i}{\sqrt2}\cosh x\,\delta_\mu{}^3\,\bar r\,,$$

$$l_\mu = \delta_\mu{}^0\,,$$

$$n_\mu = -\tfrac12\,\delta_\mu{}^0\left(1+\frac{m}{r}+\frac{\bar m}{\bar r}\right) + \delta_\mu{}^1\,.$$

Applying the following complex coordinate transformation to this tetrad

$$u' = u + i(-a\sinh x + 2\,l\cdot[\log\cosh x - \tan^{-1}\sinh x]) - 2\,l\,y\,,$$

$$r' = r + i(l - a\sinh x)\,, \qquad m' = m - i\,l\,,$$

the metric IIB of the Kinnersley [4] classification of the type {2,2} spacetimes is generated. Similar transformations on similar null tetrad forms for metrics (2) and (3) will give the metrics IIC and IID of the Kinnersley classification respectively.

Metric (4) is first written in the complex null tetrad form

$$l^\mu = \delta_1{}^\mu\,,$$

$$n^\mu = \left(r + i\,\frac{x^2}{2}\right)^{-1}\left(\bar r - i\,\frac{x^2}{2}\right)^{-1}\cdot(\delta_0{}^\mu\,r\,\bar r + \tfrac12\,[m\,r + \bar m\,\bar r]\,\delta_1{}^\mu + \delta_3{}^\mu)\,,$$

$$m^\mu = \frac{1}{\sqrt2}\left(\bar r - i\,\frac{x^2}{2}\right)^{-1}(\delta_2{}^\mu + i[-\tfrac14\,x^3\,\delta_0{}^\mu + x^{-1}\,\delta_3{}^\mu])\,.$$

The transformation

$$u' = u - \tfrac12\,i\,b\,x^2\,, \qquad r' = r + i\,b\,, \qquad m' = m + i\,,$$

when applied to this metric produces the metric IIE.

Metric (5) is first written as

$$l^\mu = (0,1,0,0)\,,$$

$$n^\mu = (1, -\tfrac12\,B\,r^2[x\,\bar x]^{-1}, 0, 0)\,,$$

$$m^\mu = \left(0, r\left[\frac1x + \frac1{\bar x}\right]\left[-\frac12\,B + \frac12\left(\frac{m}{x}+\frac{\bar m}{\bar x}\right)\right]^{\frac12}, \left[-\frac12\,B + \frac12\left(\frac{m}{x}+\frac{\bar m}{\bar x}\right)\right]^{\frac12}, i\left[-\frac12\,B + \frac12\left(\frac{m}{x}+\frac{\bar m}{\bar x}\right)\right]^{-\frac12}\right).$$

The transformation

$$x = x' + i\,a\,, \quad m = m' + i\,l\,, \quad y = 2\,y'$$

then yields the metric IVA.

* Present Address: Royal Naval College, Dartmouth.

[1] E. T. Newman and A. I. Janis, J. Math. Phys. **6**, 6915 [1965].

[2] R. P. Kerr, Phys. Rev. Letters **11**, 237 [1963].

[3] E. T. Newman and M. Demianski, Bulletin de L'Aca demie Polon. Des Sciences **11**, 65 [1966].

[4] W. Kinnersley, J. Math. Phys. **10** (1), 1195 [1969].